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MINIMUM VERTICAL SCALE SIZES IN THE WIND STRUCTURE ABOVE
100 KILOMETERS

C. O. Hines
Department of the Geophysical Sciences,
University of Chicago, U. ILL.

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UNPUBLISHED PRELIMINARY DATA

Zimmerman [1964] has presented a comparison between the small-scale wind structure, that can be detected at high altitudes, and the theoretical minimum structure sizes that can be attributed to freely propagating internal atmospheric gravity waves [Hines, 1960, henceforth Paper I]. He found that the theoretical curve adequately represented the minimum observed sizes at altitudes up to 130 km, but that, at least for the 'summer' data, it fell substantially short of the observed sizes at higher altitudes. He was led to suggest a transition to some other limiting process which would yield minimum sizes proportional to the atmospheric scale height.

While the discrepancy found by Zimmerman was in an acceptable direction, in the sense that the observed minimum scale sizes were greater than the theoretical minimum scale sizes, a re-examination of the theoretical work seemed to be in order. It became apparent that a computational or plotting error had been made by Hines in the

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preparation of Figure 12 of Paper I, that this error would lead to an incorrect estimate of the theoretical minimum scale size at a height of 225 km, and that it had in fact led (through interpolation) to a faulty theoretical curve for all heights above about 110 km in Zimmerman's diagrams.

An explicit formula for the theoretical curve is derived in the present note, and the revised curve that it yields is then compared with the data points presented by Zimmerman. The corresponding revisions that are required in Paper I have been prepared for submission to the Canadian Journal of Physics.

The theoretical curve may be defined explicitly in the following fashion. Approximations (33) and (49) of Paper I combine to yield, as the condition for a quenching of internal gravity waves,

$$2\pi\eta k_z^2 = \omega(1 - \omega^2/\omega_g^2) \quad (1)$$

where η is the kinematic viscosity, k_z is the real vertical wave number, ω is the circular frequency, and

$$\omega_g^2 \equiv (\gamma - 1)g^2/ \quad (2)$$

where γ is the specific heat ratio, g is the acceleration due to gravity, and C is the speed of sound. It can be seen from (1) that k_z^2 tends to zero as ω tends to zero or to ω_g , while a maximum occurs at the intermediate frequency

$$\omega = 3^{-1/2} \omega_g \quad (3)$$

This maximum yields in turn a minimum vertical wavelength, given by

$$\lambda_z^* \equiv 5^{3/4} \cdot 2\pi \eta^{1/2} \omega_g^{-1/2} \quad (4)$$

For numerical computations in the height range 100-200 km, where $\gamma \simeq 1.4$ and $g \simeq 9.4 \text{ m/s}^2$, this may be rewritten as

$$\lambda_z^* = 20 \eta^{1/2} H^{1/4} \quad (5)$$

where λ_z^* is measured in meters, as is the scale height $H (\equiv C^2/\gamma g)$, while η is measured in m^2/s . The approximations that lead to (1) are

valid for the purposes of this analysis provided $\lambda_z^* \lesssim H$, and such is found to be the case in the height range of present interest.

The minimum unquenched wavelength, given by (5), has been computed as a function of height from the formulae and tables of Minzner, Champion and Pond [1959]. The results, divided by two to represent half wavelengths, are shown by a solid curve marked 'FORBIDDEN-ALLOWED' in the accompanying figure. The data points presented by Zimmerman, which represent small-scale 'half wavelengths' observed in the wind structure on a variety of occasions, have been plotted on the same diagram. They are grouped under his categories 'summer' and 'winter', but are not otherwise distinguished by date or source. Also shown in the diagram, by a broken curve, is the 'FORBIDDEN-ALLOWED' transition employed by Zimmerman, derived in part from the erroneous Figure 12 of Paper I. (It departs from the new curve even at low altitudes, where it is accurate, since it represents full wavelengths rather than half wavelengths.)

The analysis given above is subject to some amendment, for it presupposes an isothermal atmosphere whereas the temperature profile of the real atmosphere plays a prominent part in determining the height variation of (5). But it seems unlikely that the error thereby introduced exceeds the uncertainties already implicit in the rather arbitrary quenching criterion, so the point will not be pursued here. It is sufficient to note for now that the curve derived from (5) provides a fair representation of the smallest vertical scale sizes observed in practice.

While there may be some transition in the limiting mechanism, as suggested by Zimmerman, the need for invoking one is now by no means so clear.

One final point should be made in passing. In his opening discussion, Zimmerman summarized the method employed by Hines in assessing the vertical scale sizes of the observed wind structure, and contrasted it with his own. While the summary fairly represents the method adopted by Hines in determining the scale size of the dominant irregular wind structure, it is not representative of the method employed in estimating minimum scale sizes. (Note that the minimum observed vertical wavelengths determined in Paper I, of 1 km at 87 km altitude and 6 km at 108 km, could not have been obtained merely by extracting a wind of constant shear and then noting successive zeros in the resultant wind profile.) Save for the fact that Hines' analysis was done by eye, and Zimmerman's by computation, no essential difference exists between their methods of estimating the small-scale structure sizes.

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References

Hines, C.O., Internal atmospheric gravity waves at ionospheric heights,
Can. J. Phys., 38, 1441-1481, 1960.

Minzner, R.A., K. S. W. Champion, and H. L. Pond, The ARDC Model
Atmosphere, 1959, Air Force Surveys in Geophysics, no. 115,
Air Force Cambridge Research Laboratories, Bedford, Mass.,
1959.

Zimmerman, S. P., "Small-scale wind structure above 100 kilometers",
J. Geophys. Res., 69, 784-785, 1964.

Legend

Comparison between the theoretical minimum vertical half
wavelengths of unquenched internal gravity waves (solid curve) and the
vertical half wavelengths observed in small-scale wind structure (data
points); see text re broken curve.

